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Procedia Engineering 127 (2015) 830 – 837

Procedia Engineering

www.elsevier.com/locate/procedia

International Conference on Computational Heat and Mass Transfer-2015

Hydromagnetic Convection in a Mushy Layer with Variable Permeability – an Analytical Study

G.S. Mytra [a*] P.K. Srimani [b]

[a] *Department of Mathematics, JSS Academy of Technical Education, Bangalore-560098, India.*
[b] *Dept of Computer Science & Mathematics, Director R&D B.U, Bangalore-560050, India*

Abstract

The problem of controlling the formation of freckles which cause imperfection during the solidification of a binary or multicomponent alloy is a challenging one. The present analytical study considers a hydromagnetic model with variable permeability near eutectic –approximation and large far-field temperature and the dynamics of the mushy layer is studied by using a modified perturbation technique. Our results suggest that although the effect of variable permeability is of stabilizing type, by a proper choice of the magnetic and expansion parameters it is possible to have an optimal control over the formation of chimney convection so that the freckles causing imperfection in the resulting solid could be drastically avoided. The profiles of marginal stability curves, vertical velocity, magnetic field, temperature and local solid-fraction are presented, which clearly predict the behavior of the system in an effective way. The results are in excellent agreement with the available results in the limiting cases.

Peer-review under responsibility of the organizing committee of ICCHMT – 2015

Keywords: Magneto-convection; Variablepermeability; Localsolid-fraction; Mushylayer; Chimneyconvection.

1. Introduction

During the solidification of a binary alloy, a distinct mushy layer made of dendritic crystals is formed due to the morphological instability of the solid-liquid interface. The partially solidified mush region is treated as a reactive porous medium. Convection that occurs during the solidification of alloys in a dendrite layer is an important phenomenon because the localized chimneys containing such flows within a mushy layer can lead to a class of defects called freckles in the resulting solid [1]. Hence, investigating the problem of convection in a mushy layer

*corresponding author Tel.+919916080622;
E-mail address: mytrashreyas@gmail.com

Peer-review under responsibility of the organizing committee of ICCHMT – 2015
doi:10.1016/j.proeng.2015.11.419

and finding the ways to control such flows are important in materials processing for producing higher quality solidified alloys. In addition there are a wide number of applications. The mushy layer forms a porous medium through which the interstial melt can flow in response to its own buoyancy in a gravitational field or to externally applied forces .The permeability of the mushy layer depends on the local liquid fraction, which in turn is determined by various internal physical processes involved in solidification. It is essential to know the permeability before any flow a melt can be analyzed, To this end, there has recently been renewed interest in measuring the liquid fraction of mushy layers in laboratory experiments [2] and [3]. Actually, there are a number of analytical studies which aimed to gain an understanding of such flows in mushy layer: for example, the studies by [4] and [5] which, were essentially based on the scaling and assumptions of the original model of [6]. In the past, investigations on magneto convection during solidification are mostly of experimental in nature. For example [7], [8], [9, 10], [11, 12], [13, 14]. All the works mentioned above are based on the case of a porous medium, modelled according to Darcy's law, with uniform permeability. It was found that the variation of permeability has a considerable effect on the convective heat transfer. The initial motivation for carrying out the present study was to uncover the effect of the vertical magnetic field in the presence of variable permeability on the chimney formation or suppression in the layer, which have important practical implications. The results of the present investigation are interesting which provide satisfactory answers to our initial motivating questions. Our results suggest that the vertical magnetic field acts as a stabilizing agent and at the same time the cumulative effect of variable permeability and the uniform external vertical magnetic field on the chimney convection is remarkable.

2. Mathematical Formulation

The physical configuration consists of a mushy layer formed during the solidification of a binary alloy that is cooled from below. The solidification front moves with a constant velocity $\vec{V_0}$in the upward direction. The whole system is under the constraint of an externally imposed magnetic field $\vec{H} = (h_1, h_2, H_0 + h_3)$ in the vertical z-direction with variable permeability effects. Here, T_e is the eutectic temperature at which the lower mush-solid interface is maintained and $T_\infty$is the temperature of the liquid far above the mushy layer. Further T_0 is the liquidus temperature of the alloy such that $T_\infty > T_0$. Here, the mushy layer is assumed to be in a state of thermodynamic equilibrium so that,

$$T = T_0(C_0) + \Gamma(C - C_0) \tag{1}$$

where, T is the temperature ,C is the composition, Γ is the slope of the liquids and C_0 is the composition of the liquid zone.

The perturbed linearzed system is given by,

The conservation of momentum

$$\mathrm{K}(\emptyset)\vec{\mathrm{q}} + \nabla \mathrm{p} + \mathrm{R}\theta\hat{\mathrm{k}} - \frac{\mathrm{Q}}{\tau}(\partial_z H) = 0 \tag{2}$$

The magnetic induction equation

$$(\partial_t - \partial_z)\vec{\mathrm{H}} = (\partial_z \vec{q}) + \frac{1}{\tau}\nabla^2\vec{\mathrm{H}} \tag{3}$$

The conservation of heat

$$(\partial_t - \partial_z)(\theta - S\emptyset) + wD\theta_B = \nabla^2\theta \tag{4}$$

The conservation of solute

$$(\partial_t - \partial_z)[(1 - \emptyset)\theta + C\emptyset] + wD\theta_B = 0 \tag{5}$$

The conservation of mass

$$\nabla\vec{q} = 0 \tag{6}$$

The conservation of magnetic field

$$\nabla\vec{H} = 0 \tag{7}$$

2.1 The Dimensionless Boundary conditions:

The dimensionless boundary conditions at the upper boundary z = d that corresponds to an impermeable (rigid) flat boundary with zero solid fraction (i.e φ=0) and at the lower boundary z=0 are

$$\theta = 0 \quad w = 0 \ \ and \ \ \vec{H} = \hat{k} \ \ at \ \ z = 0 \tag{8}$$

$$\theta = 0\,, \emptyset = 0\,, w = 0 \ \ and \ \ \vec{H} = \hat{k} \ at \ \ z = \delta \tag{9}$$

Where , $\delta = \frac{dV_0}{\kappa}$ is the growth peclet number and is the dimensionless thickness of the mushy layer.

2.2 The associated dimensionless parameters are:

$R = \frac{\beta\,\Delta C\pi(0)g}{V_0\mu}$: Rayleigh number; $= \frac{L_h}{r\Delta T}$: Stefan number ; $\tau = \frac{\kappa}{\upsilon_m}$: Roberts number.

$C = \frac{C_e - C_0}{\Delta C}$: Concentration ratio; $Q = \frac{\mu^* H_0^2 \pi(0)}{4\pi\rho_0\gamma\gamma_m}$: Chandrasekar number ; $= \frac{\pi(0)}{\pi(\emptyset)}$: Permeability function. ; T: the temperature, C_0 : Composition of the liquid zone, (u ,v, w) : Horizontal and vertical components of $\vec{q}$. $\hat{\imath}, \hat{\jmath}, \hat{k}$: Unit vectors, along the x, y, z axes. Where, the quantities have the following meaning:

$\emptyset$: 1- ψ local solid fraction ; ψ = local liquid fraction ; P :dynamic pressure;μ: is the dynamic viscosity $\pi = \pi(\psi)$: permeability is a function of the local liquid fraction ; t, T, κ, r, L_h : The Time ,Temperature , thermal diffusivity , specific heat , latent heat/unit mass; C_s : Composition of the solid phase ; C : Composition of the liquid phase; μ^*, υ_m , $\vec{H}$: Magnetic permeability , Magnetic diffusivity , Magnetic field ; ρ , ρ_0 : Densities ; $\vec{g} = (0,\ 0\,, g)$: Acceleration due to gravity;$\pi(0)$: The reference permeability.

$\vec{q} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$: Darcy velocity vector/unit area ; d : mushy layer thickness ; H_0: The strength of the magnetic field ; υ : Kinematic viscosity ; β : The volume expansion coefficient of combined heat ,mass and solute [15] and [16], $\pi(\emptyset)$: Permeability of the medium (assumed to be finite). $\pi(0)$: The reference value of the permeability in porous medium at $\emptyset = 0$.

Here, $\mathrm{w}_{00}, \theta_{00}, \emptyset_{00}, \mathrm{h}_{30}$ $\omega_{01}, \theta_{01}, \emptyset_{01}, \mathrm{h}_{301}$ are all functions of z only. Applying *curl* twice on the momentum equation and considering the *z*-component of the result we get the following system of equations in the matrix form ,after substituting (2) and collecting the terms of O($\in \delta^0$).

3. Method of solution

The method of solution constitutes two stages viz, Basic state solution and linear stability analysis.

3.1 Basic state Analysis: In the basic state, the velocity $\vec{q} = 0\,, \frac{\partial}{\partial t} = 0$ .Thus we have the following set of equations, where the subscript B indicates the basic state.

$$(1 - \emptyset_B)D\theta_B + D\emptyset_B(C - \theta_B) = 0 \tag{10}$$

$$D^2\theta_B + D\theta_B - SD\emptyset_B = 0 \tag{11}$$

$$DP_B - R\theta_B = 0 \tag{12}$$

Here, we take $\emptyset_B = \delta\emptyset_{B0}$, $\theta_B = \theta_{B0}$ (13)

on solving the differential equations, we get the following basic state solutions:

$$\theta_{B0} = A_1 + A_2 e^{-z} \quad ; \ A_1 = -1 + e\theta_\infty \ ; \ A_2 = -e\theta_\infty \tag{14}$$

$$\emptyset_B = \delta\emptyset_{B0} = A_3 + A_4 z \quad ; \qquad A_3 = -(c + s + 1) \ ; \ A_4 = e\theta_\infty \tag{15}$$

3.2 Linear stability analysis:

For performing the linear stability analysis of the perturbed system the perturbation technique through normal mode approach is employed.

The variables are expanded as follows:

$$(w,\theta,\emptyset,h_3) = (0,\theta_B,\emptyset_B,H_0)+\in\left(\widehat{w},\widehat{\theta},\widehat{\emptyset},\widehat{h_3}\right) \tag{16}$$

$$\left(\widehat{w},\widehat{\theta},\widehat{\emptyset},\widehat{h_3}\right) = [(w_{00},\theta_{00},\emptyset_{00},h_{30}) + I^*(w_{01},\theta_{01},\emptyset_{01},h_{301})]e^{ikx+\sigma t} \tag{17}$$

Where $\in \ll 1$ is the perturbation parameter, k is the horizontal component of the wave number α (here k = α) and σ is the growth rate of the disturbance. The perturbed quantities are expanded in terms of δ which is assumed to be small.

$$\mathcal{L}\alpha_{00} = 0 \tag{18}$$

where,

$$\mathcal{L} = \begin{bmatrix} (D^2-\alpha^2)+3\dfrac{D\phi_B}{(1-\phi_B)} & -\alpha^2 R_{00}(1-\phi_B)^3 & 0 & -\dfrac{Q(1-\phi_B)^3 D(D^2-\alpha^2)}{\tau} \\ -D\theta_B & D^2+D-\alpha^2-\sigma & -s(D-\sigma) & 0 \\ D\theta_B & -(1-\phi_B)(D-\sigma)+D\phi_B & (\theta_B-c)(D-\sigma)+D\theta_B & 0 \\ Dw & 0 & 0 & D-\sigma+\dfrac{1}{\tau}(D^2-\alpha^2) \end{bmatrix} \tag{19}$$

and $\alpha_{00} = [w_{00},\theta_{00},\emptyset_{00},h_{300}]^T$; T denotes the transpose of the matrix.

The solution of the first order system is given by:

$$\theta_{00} = -sin\pi z \quad ; \ \emptyset_{00} = C_1^* \, sin\pi z + C_2^* \, cos\pi z + k_1(z) \ ; \ R = R_{00} + \delta R_{01} \tag{20}$$

$$w_{00} = C_1 sin\pi z + C_2 cos\pi z + k_2(z) \, ; h_{300} = C_3 sin\pi z + C_4 cos\pi z + k_3(z) \tag{21}$$

Where,

$k_1(z), k_2(z), k_3(z)$ Determined by using the boundary conditions are given below,

$$k_1(z) = zC_2^* \quad ; \quad k_2(z) = (2z-1)C_2 \quad , \quad k_3(z) = (2z-1)C_4 \tag{22}$$

Where,

$$C_1 = \frac{A_1}{A_2}[\pi C_2^*(c+1)] \ ; C_2 = \frac{C_1^*}{A_2}\left((c+1)\right)(\pi+1) \ ; \quad C_1^* = \frac{c+s+1}{(c+s+1-e\theta_\infty)} \tag{23}$$

$$C_2^* = -\frac{(\pi^2+\alpha^2)}{\pi(c+s+1-e\theta_\infty)} \ ; \quad C_3 = -\tau\left[\frac{\pi(\pi^2+\alpha^2)C_2+\pi^2 C_1\tau}{(\pi^2+\alpha^2)^2+\pi^2\tau^2}\right] \ ; \ C_4 = \tau\left[\frac{\pi(\pi^2+\alpha^2)C_1-\pi^2 C_2\tau}{(\pi^2+\alpha^2)^2+\pi^2\tau^2}\right] \tag{24}$$

Finally from (19,20 and 21) we get the expression for R_{00} as

$$R_{00} = \frac{(\pi^2+\alpha^2)C_1}{\alpha^2 B_1^3} + \frac{3\pi C_2 A_4}{\alpha^2 B_1^4} + \frac{Q\pi(\pi^2+\alpha^2)C_4}{\tau\alpha^2} \tag{25}$$

The critical wave number α_c will be apparent in Figs. (1 to 3).The solution of the higher order inhomogeneous system is obtained by using the solutions of the basic state as well as the first-order systems, by using the solvability condition.

$$< w_{0n}, w_{00} > = \in ; < f, g > = \int f g \, dz \tag{26}$$

Now R_{01} is given by

$$R_{01} = \frac{\left[\frac{3}{4}\pi B_2 A_4 (C_1^2 - C_2^2) - \frac{3}{4} B_2 B_1^3 R_{00} \alpha^2 \left(-C_1 + \frac{C_2}{\pi}\right) - \frac{3}{4}\frac{Q}{\tau} B_2 B_1^2 \pi (\pi^2 + \alpha^2)[(C_4 C_1 - C_3 C_2) + (C_3 C_1 - C_4 C_2)]\right]}{\alpha^2 \left(\frac{C_1 B_1^3}{2} + \frac{3}{4} B_2 \left(-C_1 + \frac{C_2}{\pi}\right)\right)} \tag{27}$$

Where,

$$B_1 = 1 \;;\; B_2 = e\theta_\infty \;; A_1 = -1 + e\theta_\infty \;;\; A_2 = e\theta_\infty \;;\; \mathrm{A}_3 = -(\mathrm{c} + \mathrm{s} + 1) \;;\; \mathrm{A}_4 = e\theta_\infty \tag{28}$$

The higher order Solutions $\mathrm{W}_{01}, \theta_{01}, \emptyset_{01}$ and h_{301} are determined and the computed results are presented through graphs.

4. Results and Discussions

The computed results are presented through graphs in Figs. 1 to 7 for the experimental values of the parameters viz s=3.2,c=9.0, $\theta_\infty = 0.6,0.7,0.8$,Q=0.1,1,3,5 and $\tau = 0.0001$ respectively with $\delta = 0.01,0.03,0.05$. The expressions are quite lengthy and hence the details are not presented. In the solution process, far-field temperature $\theta_\infty$is considered at the top boundary as $D\theta_b = \theta_\infty$at $z = \delta$.The results(Figs. 1 to 3) show that, marginal stability curves are extremely sensitive to the far-field temperature .The effect of magnetic field is to stabilize the system ,as in the case of Rayleigh-Bernard convection (Fig.2) .Also, as θ_∞ increases R decreases there by indicating the destabilizing nature of θ_∞(Fig.3).The presence of the variable permeability decreases the value of R as expected. Thus chimney formation could be suppressed by enhancing the effects of uniform magnetic field. The profiles of $w = w_{00} + \delta w_{01}$, $h_3 = h_{300} + \delta h_{301}$, $\theta = \theta_{00} + \theta_{01}$, $\emptyset = \emptyset_{00} + \emptyset_{01}$ are drawn for $\delta = 0.1, 0.03,0.05$ (Figs.4to7) respectively .From Fig.4 it is observed that: (i).The effect of increase in δ is to reduce the vertical component of the velocity w.(ii).w decreases as δ increases from 0.01 to 0.05 for all z and Q. (iii).It is maximum at the middle of the layer.(iv).The variation of w with respect to z is remarkable. In the beginning increases drastically and proceeds towards left. Further, the non uniformity of the velocity profile decreases as δ increases .which clearly indicates that the effect of δ is to suppress the formation of chimney convection. Reaches maximum at z = 0.(v)But, the presence of variable permeability increases the vertical velocity as expected. For the total w=1.6346 and 7.0743 in presence and absence of variable permeability. It is found that the vertical flow decreases with increase in Q implying the stabilizing effect of the magnetic field.

From Fig.5, it is observed that total θ is negative for all and z. The values of $|\theta|$ increase as δ increases from 0.01to 0.05. As z increases from 0 to 1, the values of $|\theta|$ increase and reaches a maximum at the middle of the layer then proceeds towards right. The profile is parabolic with opening towards right (here $w = w_{00} + \delta w_{01}$); and w_{01} is a function of $\sin\pi z$ *and* $\cos\pi z$ therefore the profile of total w is parabolic in nature and the effect of δ is to increase the non-linearity/curvedness of the profile. This is in conformity with the behavior of the velocity profile.

From Fig. 6 the behaviour of total $h_3 = h_{300} + \delta h_{301}$, profile could be clearly predicted. As in the case of θ, h_3 is, (i).Negative for all z and opens towards the left. (ii).Decreases in absolute value for an increase in δ and (iii).Is maximum at z=0.6 and then decreases.

In Fig.7, the profiles of total , $\emptyset = \emptyset_{00} + \emptyset_{01}$ for $\delta = 0.01, 0.03,\ 0.05,$ are presented for Q =1.The following points are observed.(i). $\emptyset < 0$ for all z$\leq$ 0.3 and $\emptyset > 0$ for in the range $0.4 \leq z \leq 0.0$ (ii).As δ increases, the non linearity also increases. (iii).Q is maximum when z=0.7.This fig also depicts the behaviour of solid fraction in hydromagnetic convection in a mushy layer.

The present analytical work is undertaken and the mathematical model is presented along with the necessary assumptions in order to predict the cumulative effect of uniform vertical magnetic field and variable permeability by applying a modified perturbation technique which is found to be very elegant.

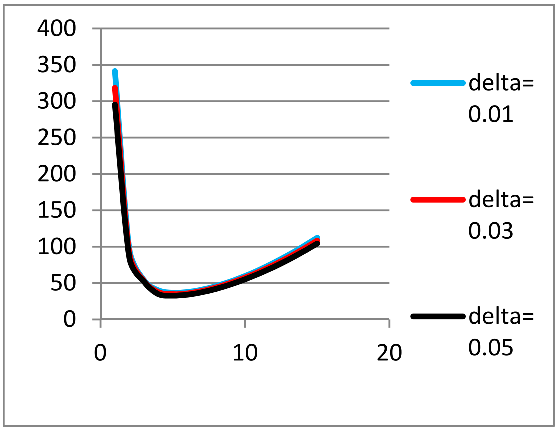

Fig.1. R v/s Z for $\delta =$ 0.01 ,0.03, 0.05.

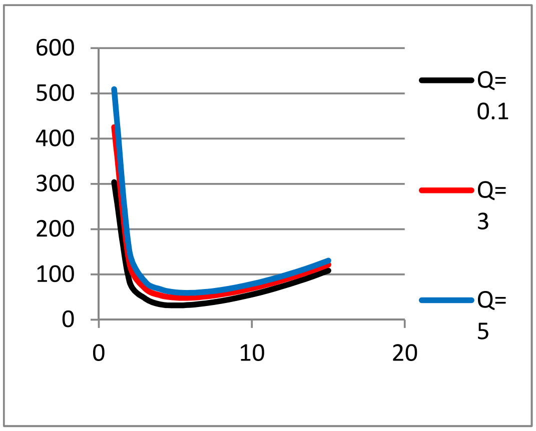

Fig.2. R v/s Z for Q = 0.1, 3, 5.

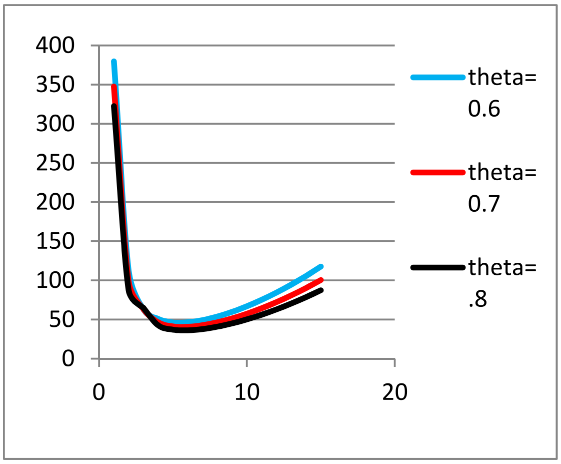

Fig.3. R v/s Z for $\theta_{\infty} =$ 0.6, 0.7, 0.8

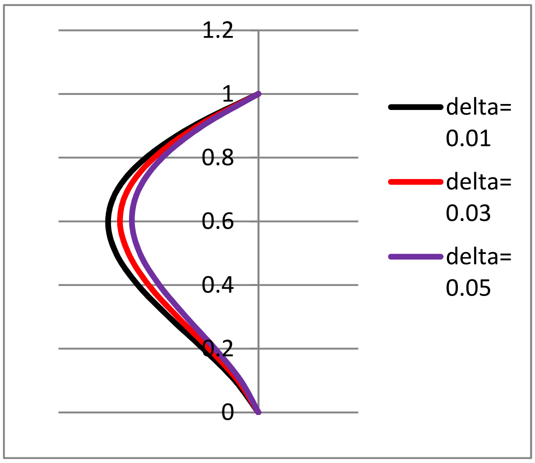

Fig.4.W v/s Z for Q=1

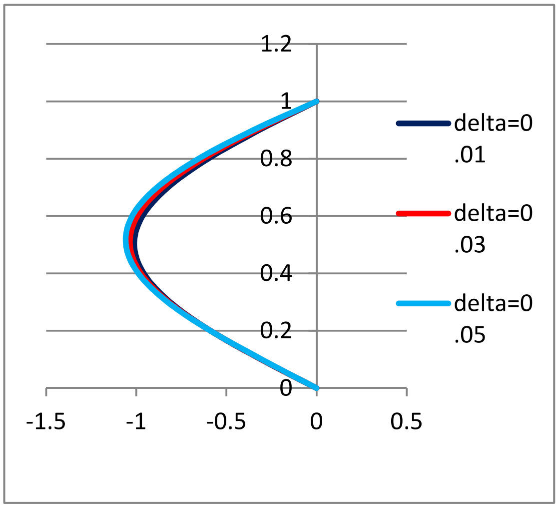

Fig.5 θ v/s Z for Q=1

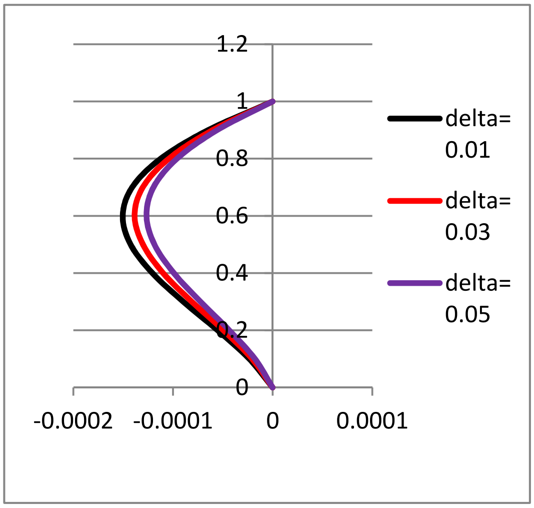

Fig.6. h_3 v /s Z for Q=1

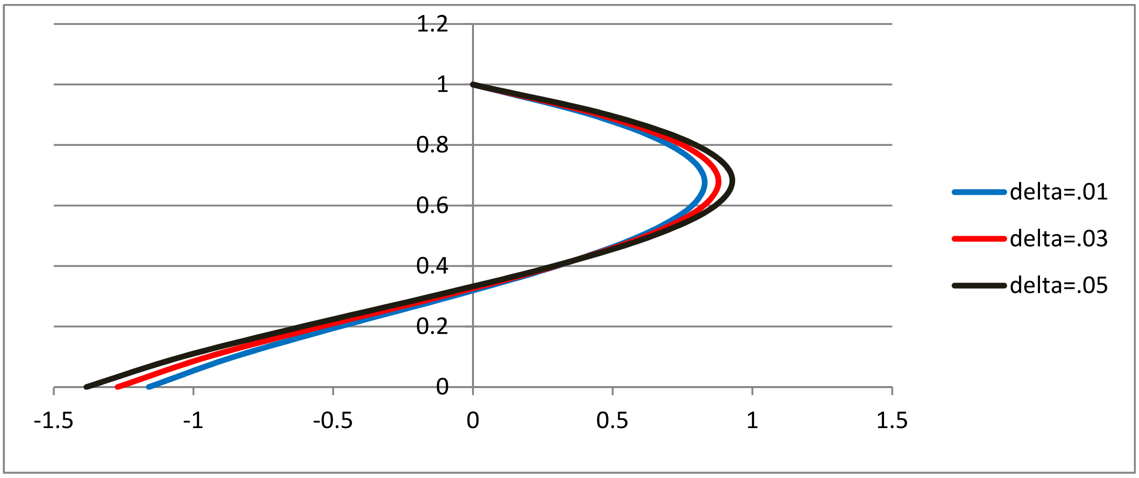

Fig.7. $\emptyset$ v/s Z for Q = 1

Through the present analytical approach which is not available in the literature it is shown that the complete solution of the basic , first and the higher order systems could be accurately determined and finally it is concluded that, a proper choice of the variable permeability and magnetic parameters it is possible to have a complete rather an optimal control over the formation of chimney due to the morphological instability at interface during the solidification process of a binary alloy, which in turn results in a solid free from freckles. It is important to note that our model is best suited for controlling convection in a mushy layer for (i). Large far-field temperature.(i.e., $\theta_\infty = 0.6,0.7,0.8$). (ii). Uniform magnetic field. (iii).Variable permeability, are considered. Actually, large far-field temperature inhibits the formation of chimney convection See [10].

5. Acknowledgment

One of the authors Mrs. G. S. Mytra acknowledges J. S. S. Academy Of Technical education, Bangalore, Karnataka and Bharathiar University, Coimbatore, Tamilnadu, India for providing the facility to carry out the research work.

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